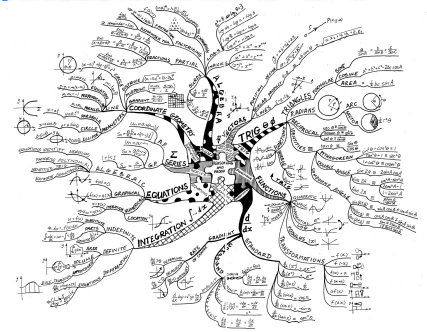


**“MATHEMATICS ARISES FROM THE HUMAN MIND AS IT COMES INTO CONTACT WITH THE WORLD AND AS IT CONTEMPLATES THE UNIVERSE AND THE FACTORS OF TIME AND SPACE.”**

MARIA MONTESSORI



**B**efore we are even born into this world, our existence is recorded with numbers; once we are born, numbers continue to track our existence, marking the minutes, days, and years spent on Earth and even long after we’ve left it, numbers mark a time of what once was. The idea of ‘number’ is one of the most abstract concepts the human mind encounters and our understanding [of number] grows with our experiences in society and the complexity of our needs.

Maria Montessori recognized that children are born with a particular kind of mind, one that is naturally inclined towards order. This special mind is what gives humans the ability to make judgments and to calculate; it is how we have progressed in fields such as engineering and architecture. Montessori called this ‘the mathematical mind’ - a term borrowed from the French physicist and philosopher Blaise Pascal. This part of the mind is vital in one’s understanding of the world. Thus, the study of mathematics provides intellectual preparation for innovative and critical thought, as well as helps to develop the ‘mathematical mind’.

**MATHEMATICS**

“The abstract science of number, quantity, and space, either as abstract concepts (pure mathematics), or as applied to other disciplines such as physics and engineering (applied mathematics)”

**Oxford Dictionary**

Even without conscious thought, the mind ‘tends to estimate and quantify, to see identity, similarity, difference, and patterns, to make order and sequence, and to control error’. This very truth shaped Montessori’s philosophy of how mathematics should be

presented to the child. She knew that the 'hand is the teacher of the mind'. Indeed, it is 'the hand that constructs the neurological pathways to the brain'; and so Montessori took a most abstract skill and brought it to life.

Mathematics comes to life in the Montessori classroom with concrete materials that are beautiful yet simple. The materials give the child a 'sensorial experience of the abstraction that is mathematics, allowing them to store concepts so that when the time comes to deal exclusively in abstract terms, the understanding is already there'. The child learns math effortlessly through hands-on manipulation, movement, and through engaging his senses. A prime example is the Number Rods; the material used to introduce the concept of quantity. These wooden rods are painted in sections of red and blue so that each section represents the addition of a unit. For that reason, the rod for two is twice as long and twice as heavy as the rod of one, the rod for ten is ten times larger than that for one, thus making the concept of quantity extremely apparent. Furthermore, counting individual objects such as beads requires the child to make the additional mental step of grouping objects together in order to come up with the quantity.

The Montessori approach offers another concrete experience to introduce the Decimal system in the form of the Golden Bead material. The child can clearly see the differences between one, ten, one hundred and one thousand: one unit is represented with one golden bead whereas one thousand is a cube made up of one thousand golden beads. This vision strengthens their confidence and knowledge of quantity, serving as a basis of understanding before moving on to materials that are more abstract representations: the Snake game, the Stamp game, the Small and Large Bead Frame, the Checkerboard and the Hierarchical material.

Montessori designed the mathematics materials in such a way that they incorporate the 'spontaneity and natural capabilities of a child's mathematical mind'. All of the Montessori math materials are designed with their own 'control of error'; thus allowing the child to assess his own progress as he develops his mind. He is introduced to the concepts of addition, subtraction, multiplication and

division in activities where he is given an actual concrete experience of the meaning of these functions. For example, he experiences addition as the putting together of two quantities that results in the production of a larger quantity and multiplication as a special addition in that it is the putting together of quantities that are all the same.

Montessori observed that the child possesses a mind 'capable of effortlessly taking in unlimited information and so the more frequent his experiences, the deeper they are imprinted in his subconscious'. Colored bead chains allow the child to count with linear progression or by using skip counting. The Snake game and Stamp game show patterns with operations, leading the child to discover the basic and essential combinations, the commutative law, as it applies to addition and multiplication, and the relationships in subtraction and division. Children enjoy repeating these engaging activities and will choose to do so all while strengthening their memorization abilities. The hands of the child teach him as he discovers the ability to calculate and create a world of his own.

**T**he study of mathematics in the Montessori classroom is made up of many little details that form a whole, but each detail is complete unto itself. This correlates with the idea of interconnectedness amongst all things, the underlining message of the Montessori philosophy. There is a beauty in the knowledge of arithmetic in that it is *of the universe*. While Language can be distinct to a particular people, Mathematics speaks universally. The rules of arithmetic are uniform in every language. It is through one's understanding of mathematics that he is able to manipulate and understand his world. Such understanding then paves the road for extraordinary creations to emerge and be level with our evolving existence.

*Trace the memorization skills of one of the four operations (addition) from concrete materials to abstraction.*

**“OUR AIM IS NOT ONLY TO MAKE THE CHILD UNDERSTAND, AND STILL LESS TO FORCE HIM TO MEMORIZE, BUT SO TO TOUCH HIS IMAGINATION AS TO ENTHUSE HIM TO HIS INNERMOST CORE.”**

MARIA MONTESSORI

**T**o memorize means to learn by heart and to fix experience and knowledge in our memory. A child can memorize information in two ways. One way is to learn something mechanically and by rote. The second way is to prepare and organize a variety of activities all having the same aim that the child can work through. This variety of activities keeps the child’s imagination alive. Maria Montessori developed many different exercises for the memorization of facts for elementary students. This repetitive practice allows the child to memorize their math facts in an engaging way.

Memorizing math facts is also a developmental skill. Children should only begin memorizing math facts once they have a solid foundation in the operation at hand. In the Montessori classroom, concepts are presented, practiced with concrete materials and spiraled throughout the curriculum. This means, each year concepts are explored in more depth and practiced with materials that grow increasingly abstract, as the child is able to think more abstractly. Montessori recognized the power of the mathematical mind in human development and created tools to help children explore the keys to mathematical thinking. All these tools are made for clearing the concept of mathematics more logically for the children.

The memorization skills of addition begin with the Golden Bead material. The child gathers all of the beads of the equation in question and physically combines the addends together, making a sum. It is apparent to the child that when two smaller quantities come together, a larger quantity is formed. The golden beads allow a child to clearly see that one is smaller than ten, ten is smaller than 100, and 100 is smaller than 1000. The child sees that ten unit beads make up one ten bead bar; ten bead bars make up one hundred square; and that that ten hundred squares

make up one thousand cube. The weight of these can also be compared, allowing for a strong sensorial connection, the greater the quantity, and the heavier the weight. The child also has access to various addition charts and the addition strip board to memorize his addition facts. As the repetition and use of materials imprints in their mind, the memorization begins and addition becomes simpler; the materials serve as a control of error to confirm math done mentally.

When the child has worked through the golden bead material, he moves on to the stamp game. This is one step closer to abstraction. Colors represents the place values: units are green, tens are blue, hundreds are red; the color scheme then repeats as the thousands is actually the ones, the ten thousands are tens and so on. The Stamp game material still allows the child to count concretely however, because the stamps all the same size regardless of the quantity it represents, he is transitioning to abstraction: each different colored tile represents a different place value. The child then transitions to using the Dot game. This is not a concrete material, rather a practice of making dots on a board, where the colors relate to the place value, and counting [those dots] up.

**M**aria Montessori often quoted the French philosopher and mathematician, Pascal, who said that all humans have a “mathematical mind”. She believed this to be especially true of children. Children explore the world by organizing and categorizing what they find there and they love to find the patterns in the world around them. This marks the beginning of their mathematical education.

**“HUMAN INTELLIGENCE TODAY IS NO LONGER A NATURAL INTELLIGENCE BUT A MATHEMATICAL INTELLIGENCE. WITHOUT A MATHEMATICAL EDUCATION IT IS IMPOSSIBLE TO UNDERSTAND THE PROGRESS OF OUR TIME OR TO PARTICIPATE IN IT. IN OUR TIME, A MIND WITHOUT MATHEMATICAL CULTURE IS COMPARABLE TO THAT OF A [PERSON] IGNORANT OF THE ALPHABET... IN ITS NATURAL STATE THE HUMAN MIND IS ALREADY MATHEMATICAL: IT TENDS TOWARD EXACTNESS, MEASURE AND COMPARISON.”**

MARIA MONTESSORI

**"THERE WAS NO METHOD TO BE SEEN, WHAT WAS SEEN WAS A CHILD...ACTING ACCORDING TO ITS OWN NATURE."**

MARIA MONTESSORI



Sharing is a characteristic that emerges with age. This developmental aspect [of sharing] becomes clear in the lower elementary classroom, as the children willingly share and thrive from teamwork. This presents a fitting time to embark on the mathematical concept of fractions.

The study of fractions in the Montessori curriculum, like all other areas, is presented through concrete experiences and progressively moves to abstract representations. The fraction materials provide the sensorial experience in that the child is able to feel and see the fraction representations and use [the materials] to discover for himself concepts such as equivalent fractions.

**FRACTUS**  
(Latin)  
means "to break, cut or divide".

Fraction skittles are used to sensorially introduce the concept of fractions and to represent divisors of whole, one-half, one-third, and one-fourth in division equations. The fraction circle insets introduce the child to the concept of quantity of fractions and all aspects of fraction work: proper terminology, equivalency, and the arithmetic functions, as well as measurement of angles in geometry. In his initial introduction, the child is presented lessons using the fraction circle insets and works with them to learn the fraction families and their corresponding symbols. He is then able to work

independently using the box of plastic fraction pieces, which are labeled for each of the values from halves to tenths.

Using the fraction materials as well as understanding various terms such as “families” to describe denominators and the concept of division as “sharing equally”, the child can explore a variety of concepts. In this way, the child is able to isolate the concept in question while being able to understand how each concept relates to another. The child will then discover how to perform all addition, subtraction, multiplication and division problems using fractions and whole numbers. The child will work on a sequence of fraction lessons throughout his time in the lower elementary classroom and be ready to learn more advanced concepts as he continues on in the upper elementary classroom.